Warp-Drive Spacetime

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In this short paper we will investigate a theoretical method where we attempt 'faster' than light travel. Note that I will be using natural units for simplicity ($c = 1$). Consider a curve $x = x_s(t)$, $y = 0$, $z = 0$, lying on the $t - x$ plane passing through the origin. The line element specifying the metric is

$$ds^2 = -\,dt^2 + [dx - \dot{x}_s(t)f(r_s)\,dt]^2 + dy^2 + dz^2 \tag{1}$$

and $r_s \equiv (x - x_s(t))^2 + y^2 + z^2$. The function $f(r_s)$ is any smooth positive function that satisfies $f(0) = 1$ and decreases away from the origin and goes to zero when $r_s > R$ for some value of R. It is evident that spacetime is flat where $f(r_s)$ vanishes, but curved where it does not. The light cones at a point in the $t - x$ plane are curves emerge from the point with $ds^2 = 0$ (null separation), that is, with

$$ds^2 = -\,dt^2 + [dx - \dot{x}_s(t)f(r_s)\,dt]^2 \tag{2}$$

and rearranging,

$$\frac{dx}{dt} = \pm 1 + \dot{x}_s(t)f(r_s) \tag{3}$$

which describe the light cone in this 2 dimensional spacetime diagram. Inside regions where spacetime is flat ($f(r_s) = 0$), the light cones are the usual 45° lines. However, inside regions where we have curved spacetime ($f(r_s) \neq 0$), the light cones are tipped over. To see the implications of this, consider a spaceship travelling

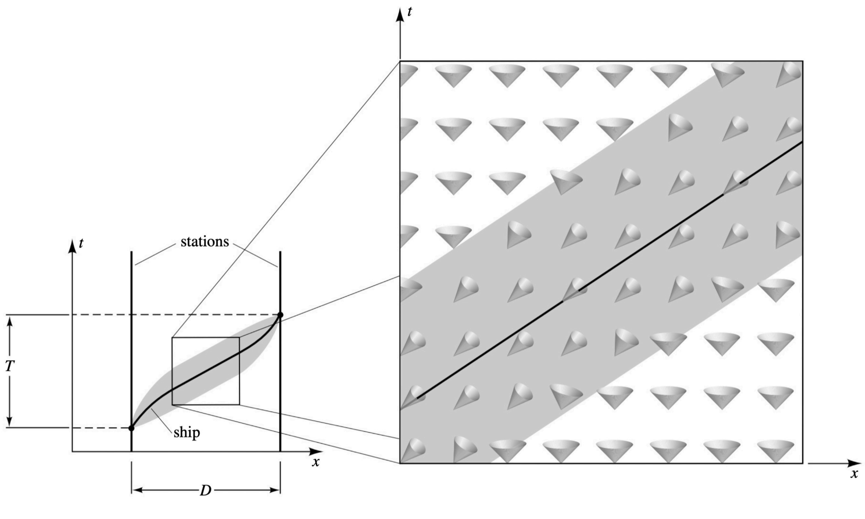

Figure 1: Light cones in warp-drive spacetime: shaded regions indicate curved spacetime and non-shaded regions indicate flat regions.

along the curve $x_s(t)$ between two space stations in an elapsed time of $\tau < D$ (remember the use of natural units). It looks like the spaceship has travelled faster than the speed of light. Indeed, such a curve requires $\dot{x}_s(t) = D/\tau > 1$ somewhere. If the spacetime is flat for the curve the spaceship travels, the spaceship would be moving at greater than the speed of light. But the spacetime is not flat and the light cones are tipped over, the curve is still inside the time-like region of every light cone along the curve. Hence the spaceship is always moving at less than the local velocity of light, despite the coordinate velocity $\dot{x}_s(t)$ is sometimes greater than 1.

This means that for an observer in the flat space outside who knows nothing of the curvature bubble, the ship would have traversed a distance that would be impossible without faster than light travel. If the human civilisation could somehow construct a spaceship that warps a region of spacetime curvature to the form of Equation 1 with $f(r_s) \neq 0$, we would have what is known as a "warp-drive", enabling travel across the galaxy in times much less than that taken by light itself!